سنگی با سرعت v_0 به طرف بالا پرتاب می‌شود.در لحظه عبور از پنجره A سرعت v_1 و در لحظه عبور از کنار پنجره B که 3 متر بالاتر از A است، سرعت V_2 دارد.بیشینه ارتفاع سنگ نسبت به پنجره B را بیابید اگر $V_1=2V_2$

A) 1 m
B) 2 m
C) 3 m
D) 4 m

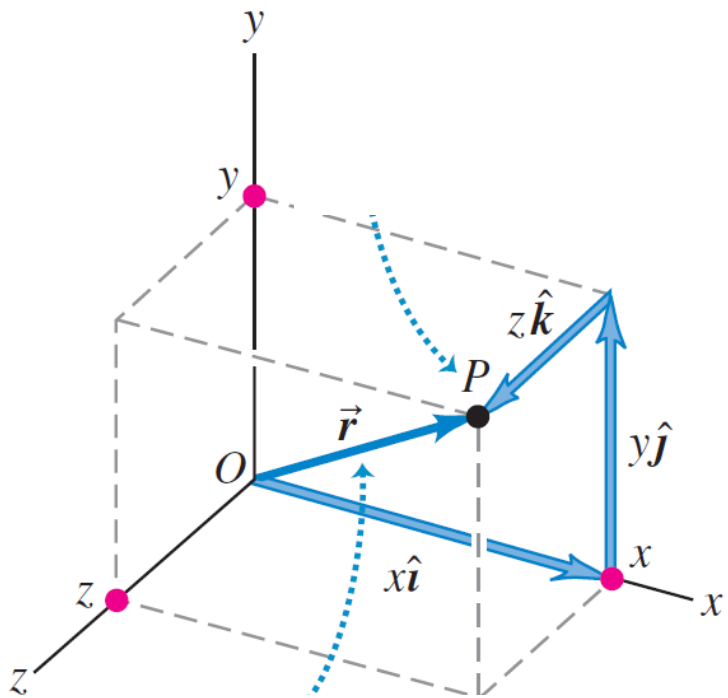
سوال) خودروی a با سرعت ثابت 10 m/s در حال حرکت است. خودرو b در لحظه عبور خودروی a از کنار آن، از حالت سکون با شتاب ثابت $2\ m/s^2$ شروع به حرکت می‌کند. بعد از گذشت چه زمانی خودروی b به a می‌رسد؟

a) 5

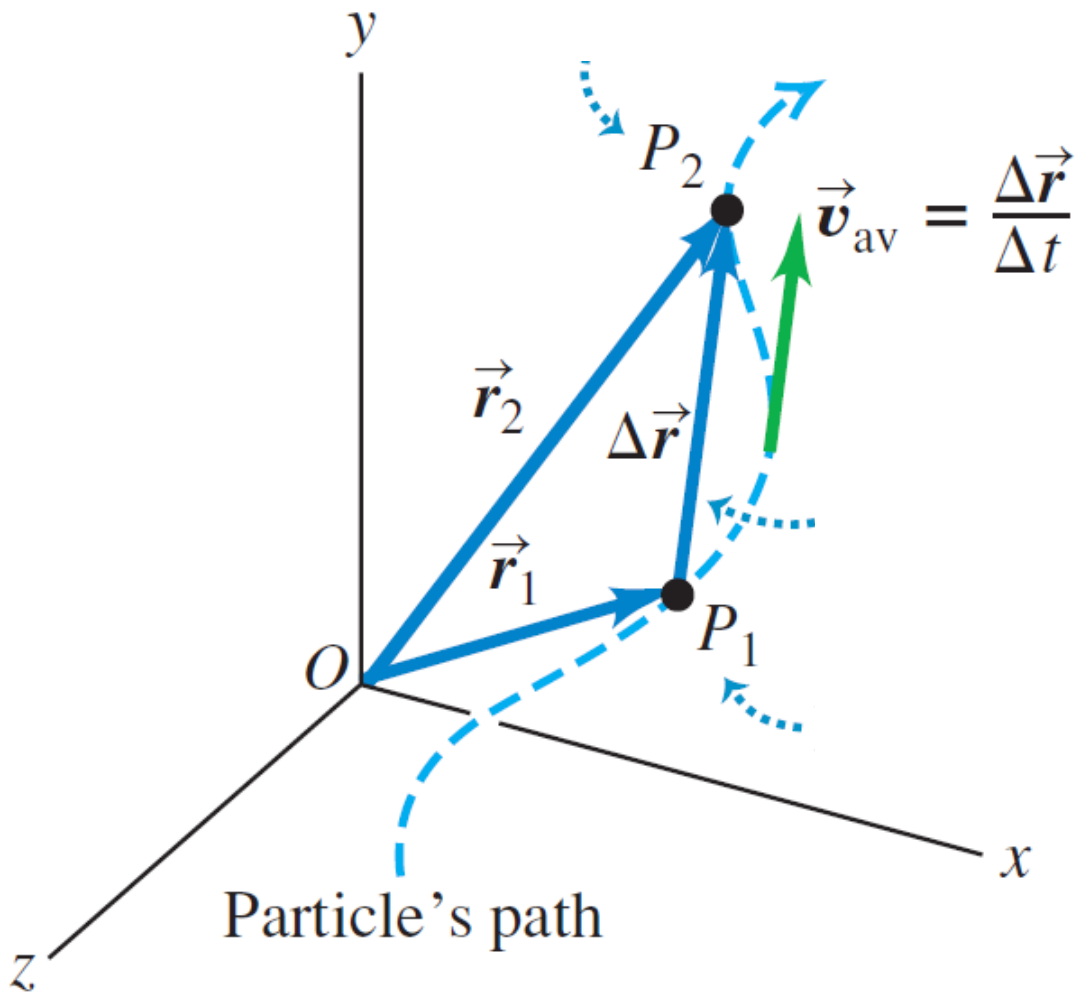
b) 10

c) 15

d) 3



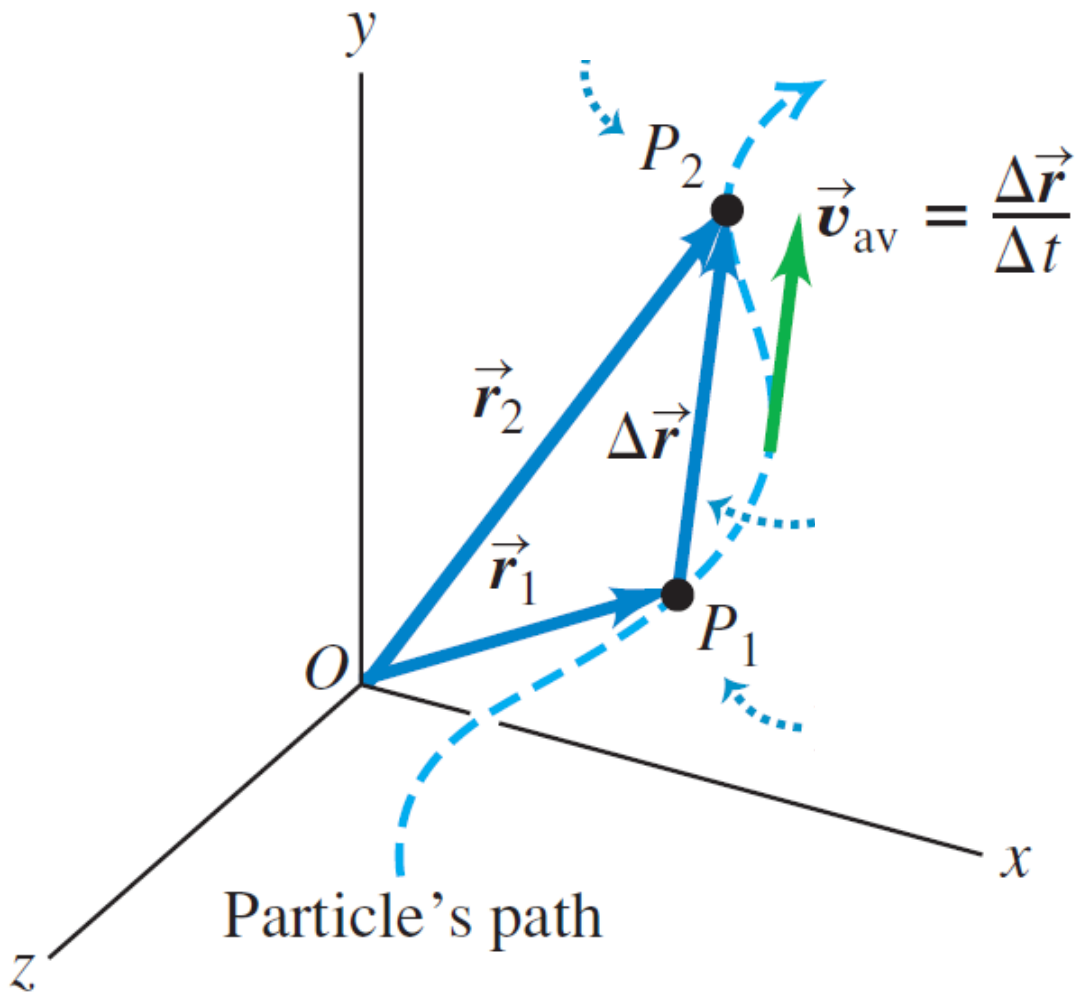
$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$



$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1.$$

$$\Delta r = (x_2\mathrm{I} + y_2\mathrm{J} + z_2\mathrm{K}) - (x_1\hat{\mathrm{i}} + y_1\hat{\mathrm{j}} + z_1\hat{\mathrm{k}})$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{\mathrm{i}} + (y_2 - y_1)\hat{\mathrm{j}} + (z_2 - z_1)\hat{\mathrm{k}}$$



$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

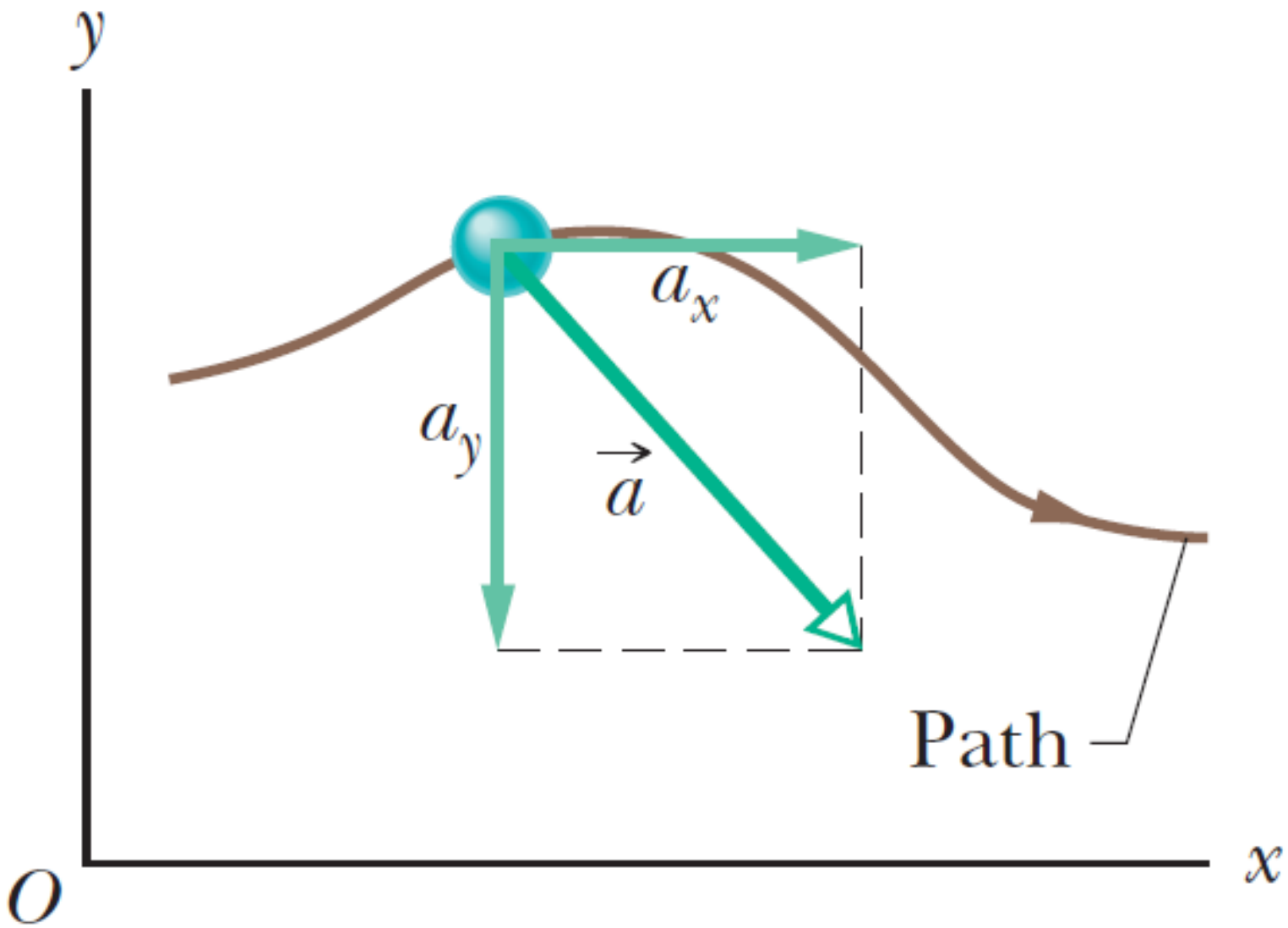
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{a}_{\text{avg}} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}.$$

$$\vec{a} = \frac{d\vec{v}}{dt}.$$

$$\vec{a} = \frac{d}{dt}(v_x\hat{\mathrm{i}} + v_y\hat{\mathrm{j}} + v_z\hat{\mathrm{k}})$$

$$= \frac{dv_x}{dt}\hat{\mathrm{i}} + \frac{dv_y}{dt}\hat{\mathrm{j}} + \frac{dv_z}{dt}\hat{\mathrm{k}}.$$


y
a_x
a_y
$\vec{a}$
Path
O
x

مثال

$x = -3t^3 - 4t \quad \text{and} \quad y = -5t^2 + 6$

$v_x = -9t^2 - 4$ $\qquad$ $v_y = -10t$

$a_x = -18t$ $\qquad$ $a_y = -10$

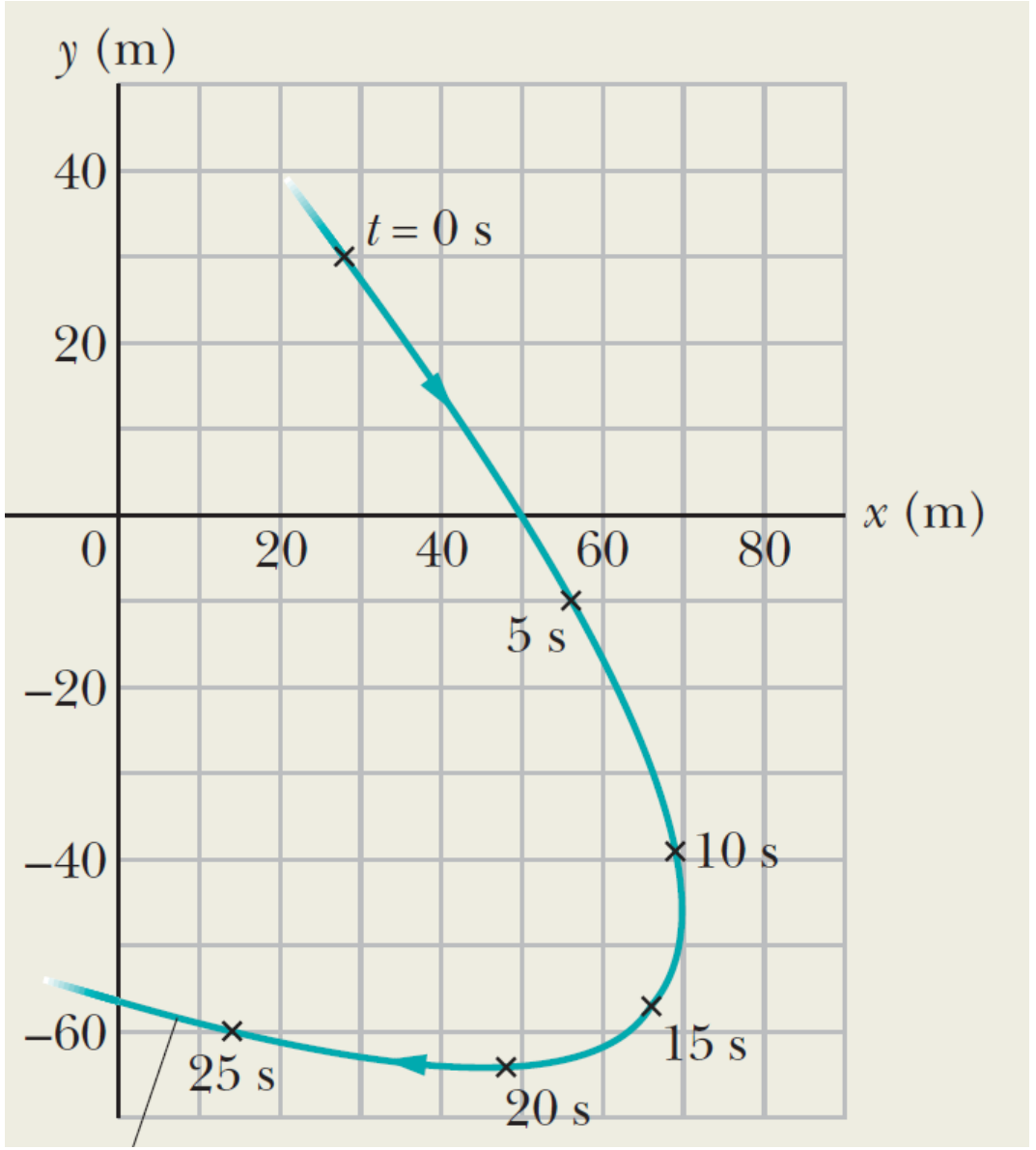
$$\left|\vec{v}\right| = \sqrt{{v_x}^2 + {v_y}^2} \qquad \tan\theta = \frac{v_y}{v_x}$$

مثال. یک خرگوش بر روی یک مسیر با معادله زیر در حال حرکت است. در لحظه t=15 s مکان، سرعت و شتاب خرگوش را بیابید.

$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

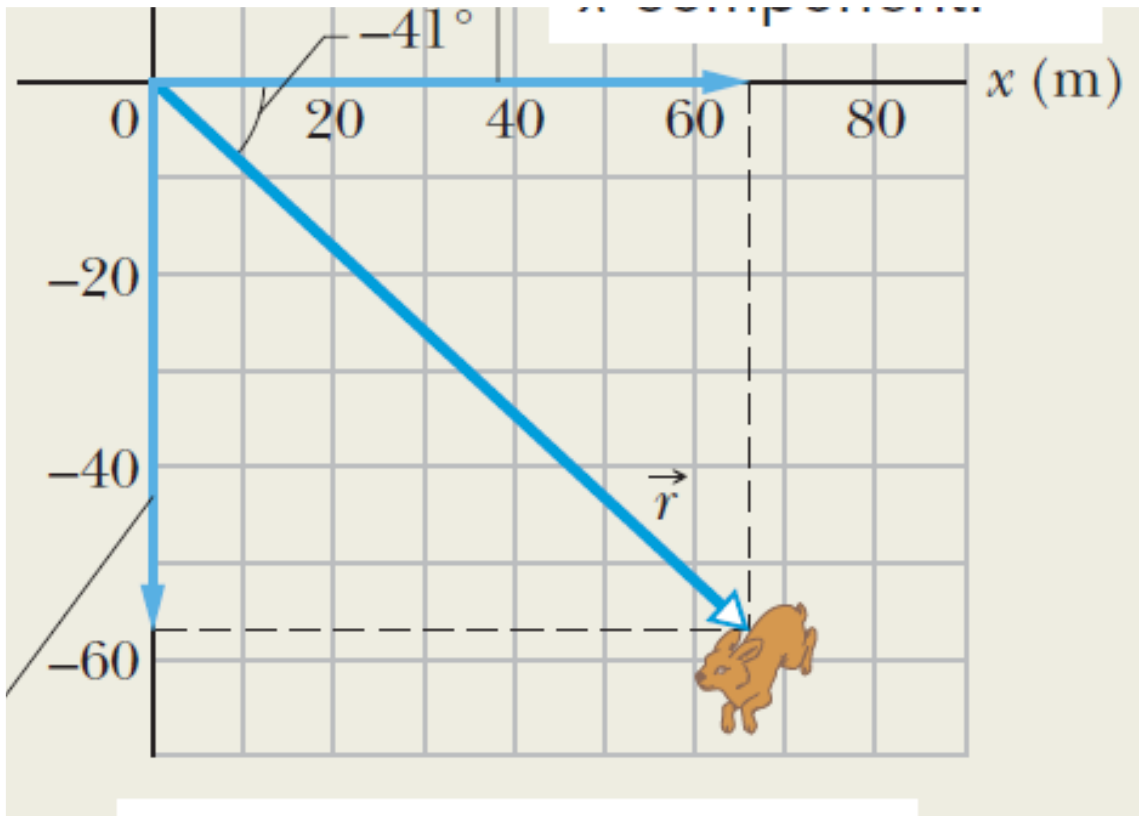
$$\vec{r}(t) = x(t)\hat{\mathrm{i}} + y(t)\hat{\mathrm{j}}.$$

$$\vec{r} = (66\ \mathrm{m})\hat{\mathrm{i}} - (57\ \mathrm{m})\hat{\mathrm{j}},$$



$$r = \sqrt{x^2 + y^2} = \sqrt{(66\ \text{m})^2 + (-57\ \text{m})^2}$$
$$= 87\ \text{m},$$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-1}\left(\frac{-57\ \text{m}}{66\ \text{m}}\right) = -41°.$$

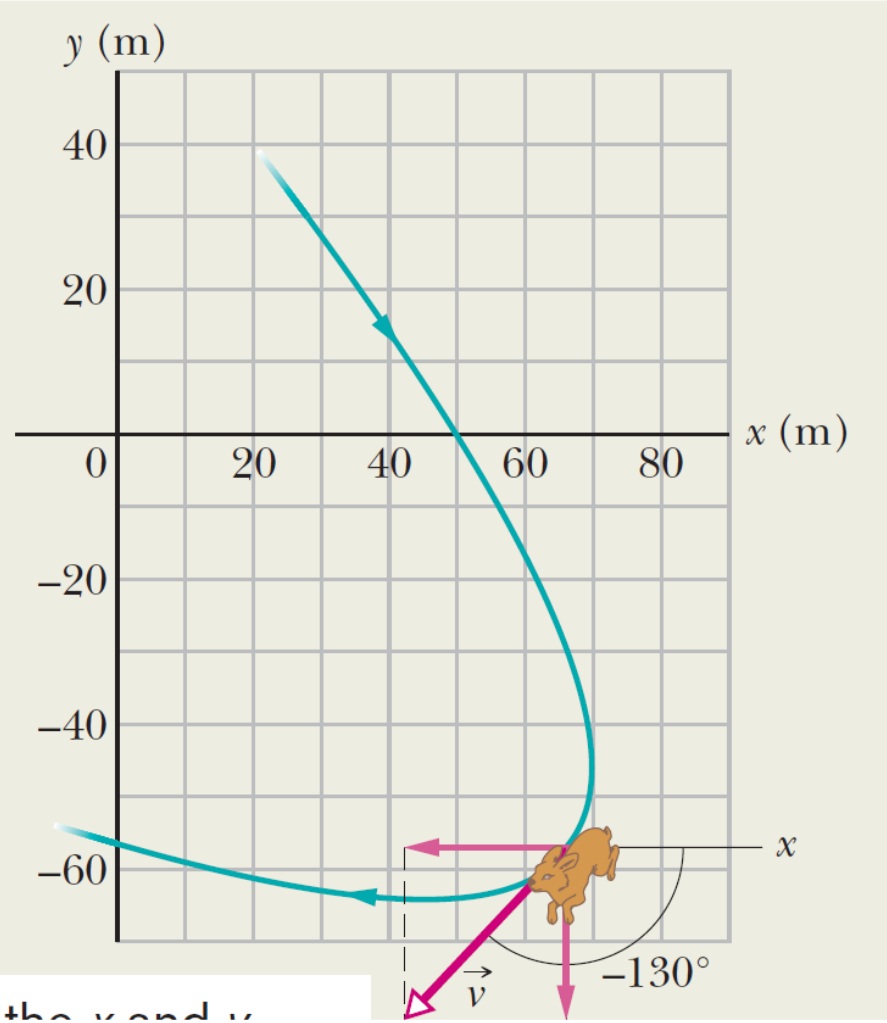


$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(-0.31t^2 + 7.2t + 28)$$
$$= -0.62t + 7.2.$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt}(0.22t^2 - 9.1t + 30)$$
$$= 0.44t - 9.1.$$

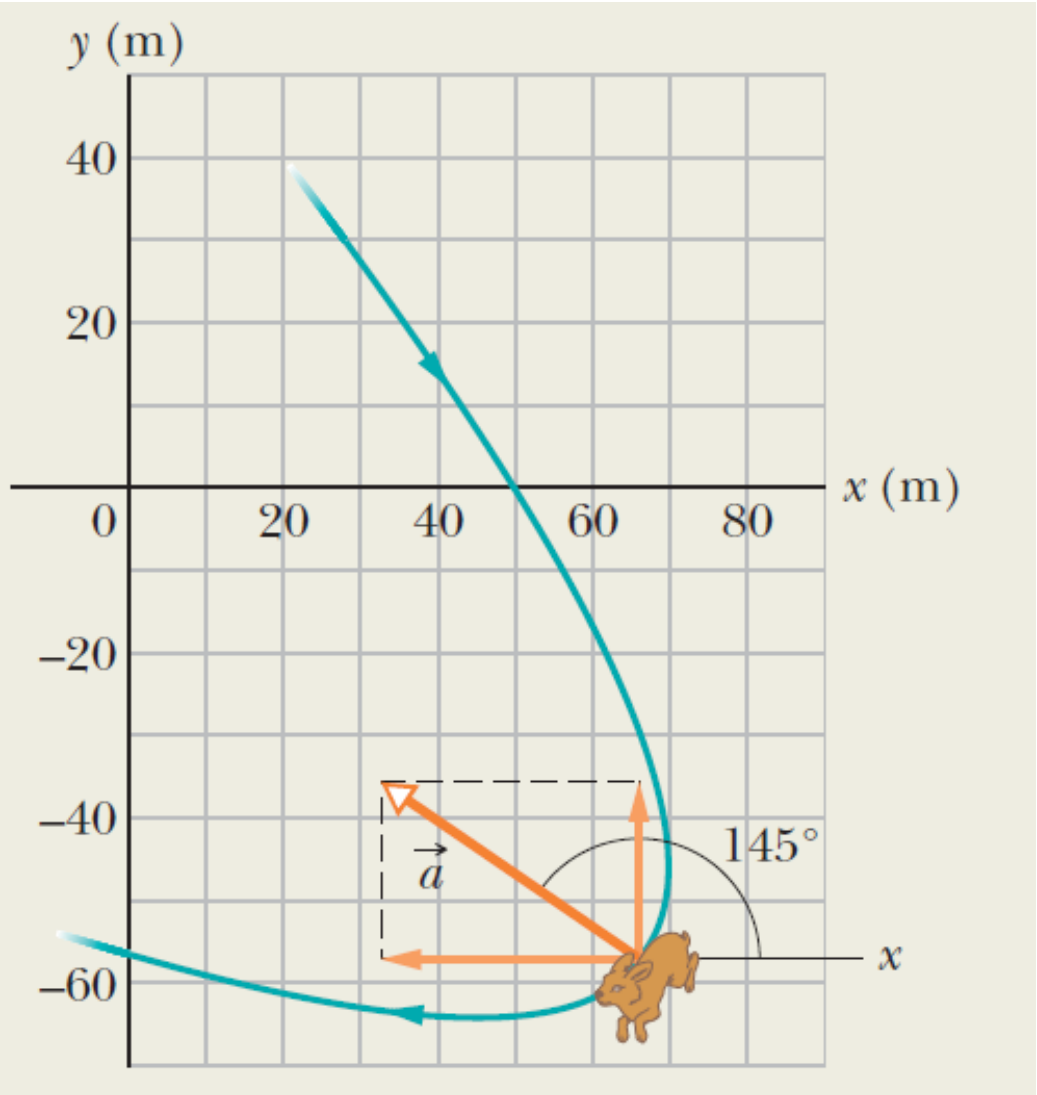
$$\vec{v} = (-2.1\ \text{m/s})\hat{i} + (-2.5\ \text{m/s})\hat{j},$$

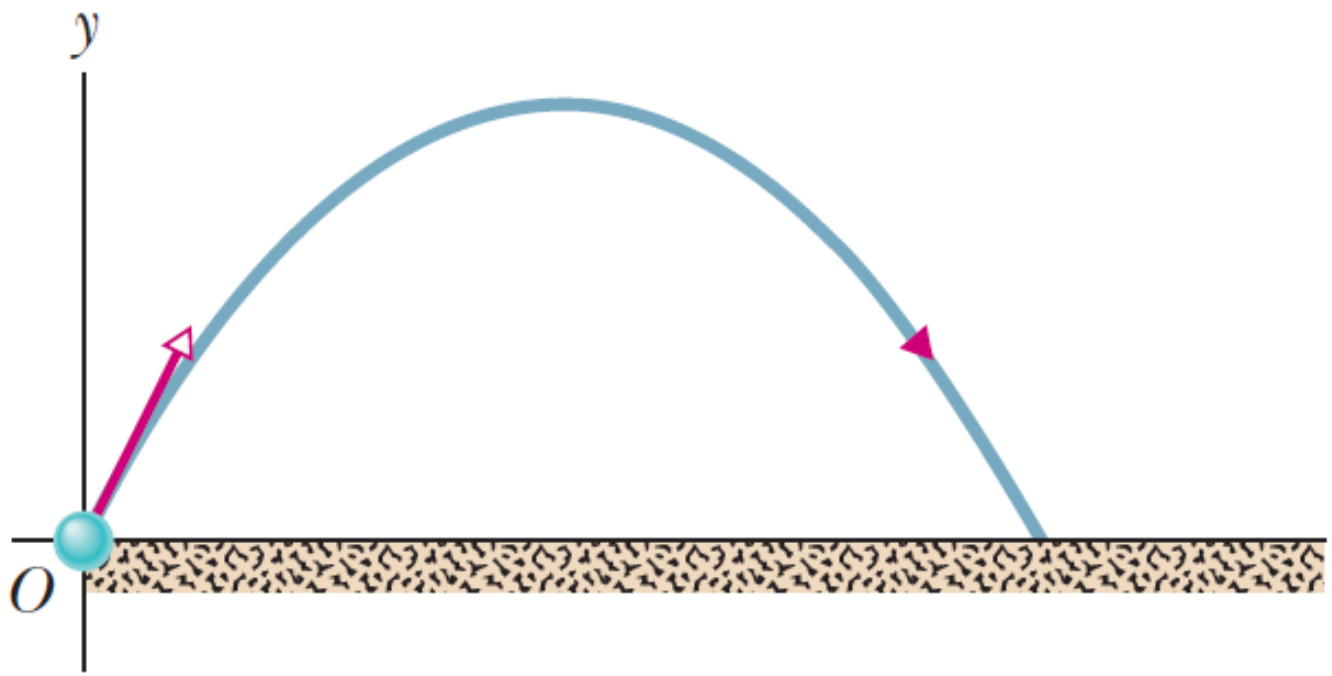


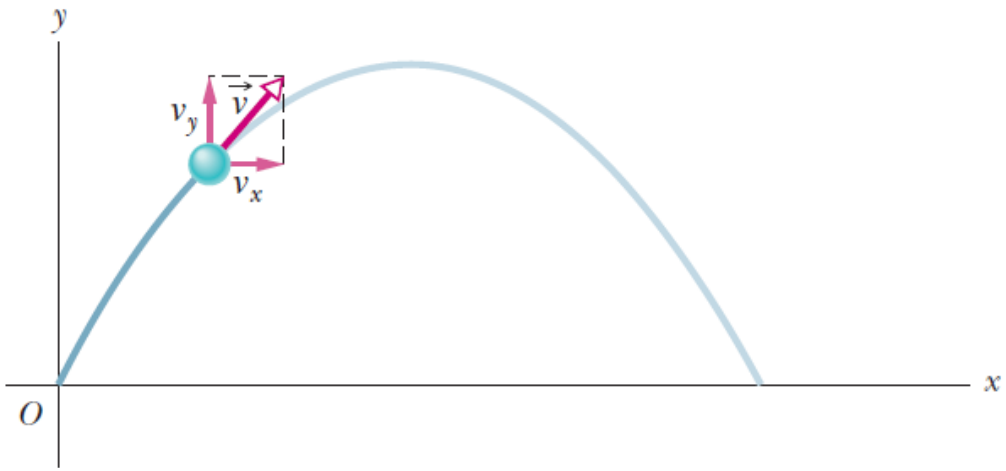
$$x = -0.31t^2 + 7.2t + 28$$
$$y = 0.22t^2 - 9.1t + 30.$$

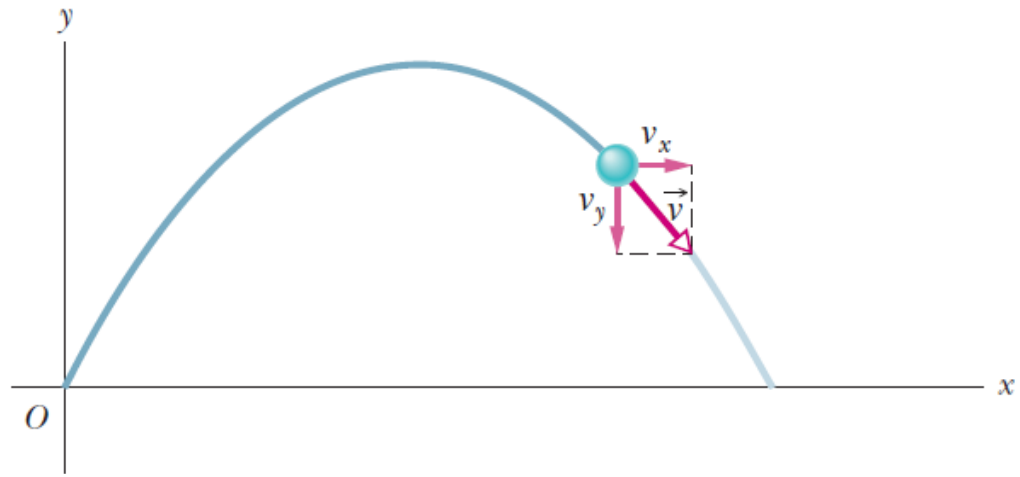
$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}(-0.62t + 7.2) = -0.62\ \text{m/s}^2.$$

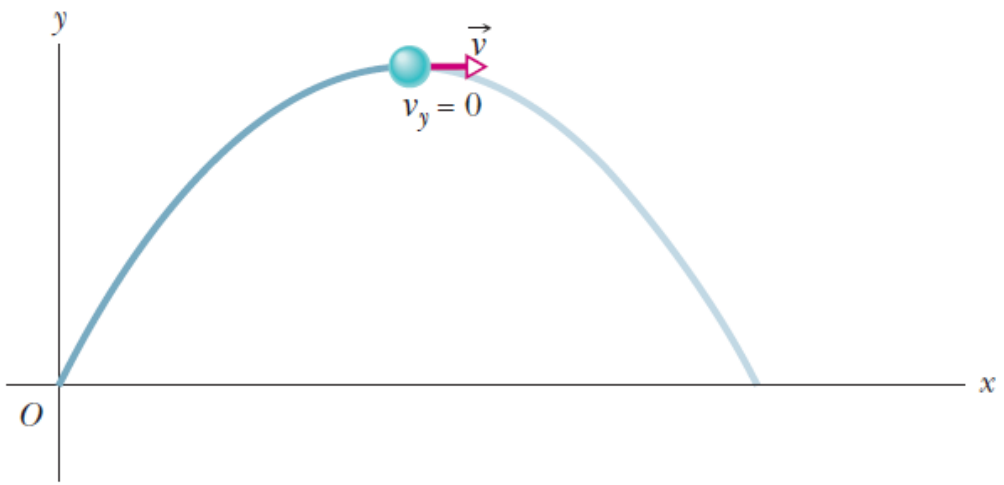
$$a_y = \frac{dv_y}{dt} = \frac{d}{dt}(0.44t - 9.1) = 0.44\ \text{m/s}^2.$$

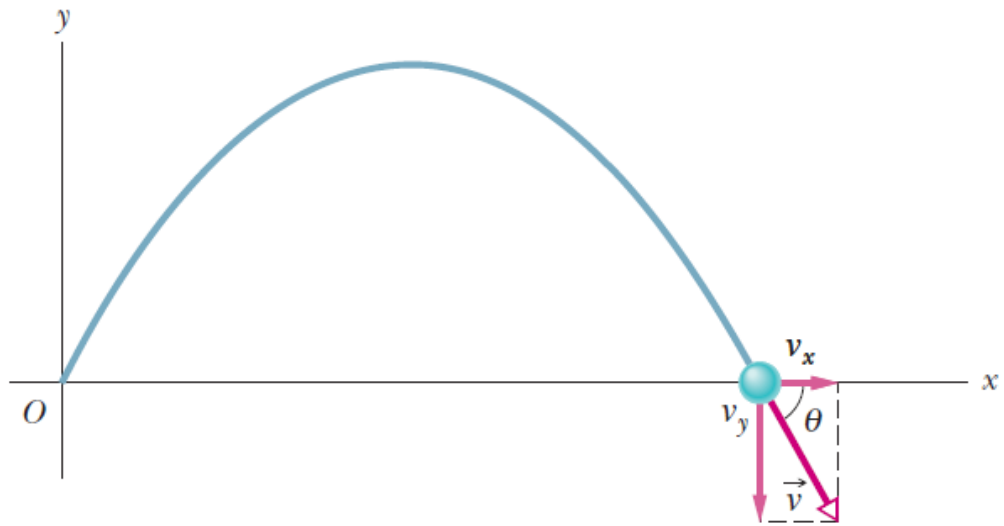


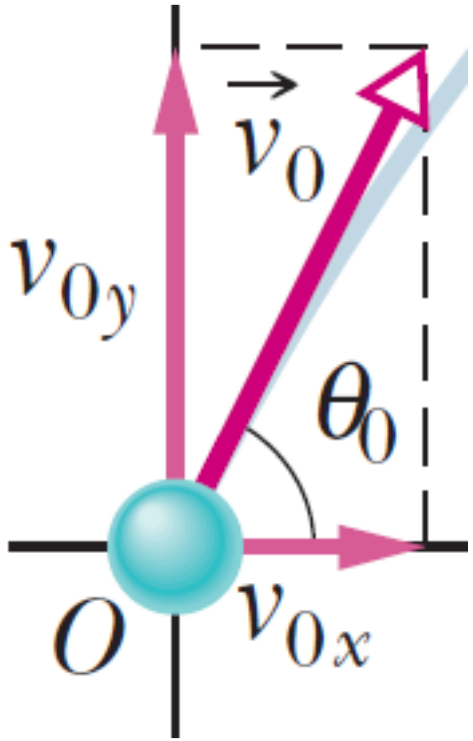

y
O


y
v_y
$\vec{v}$
v_x
O
x


y
v_x
v_y
$\vec{v}$
O
x


y
$\vec{v}$
$v_y = 0$
O
x


y
v_x
v_y
θ
$\vec{v}$
O
x



$$v_{0x} = v_0 \cos \theta_0$$

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

$$= (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2,$$

$$a = 0$$

$$x - x_0 = v_{0x}t.$$

$$x - x_0 = (v_0 \cos \theta_0)t.$$

$$R = (v_0 \cos \theta_0)t$$

$$0 = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2.$$

$$R = \frac{2v_0^2}{g} \sin \theta_0 \cos \theta_0.$$

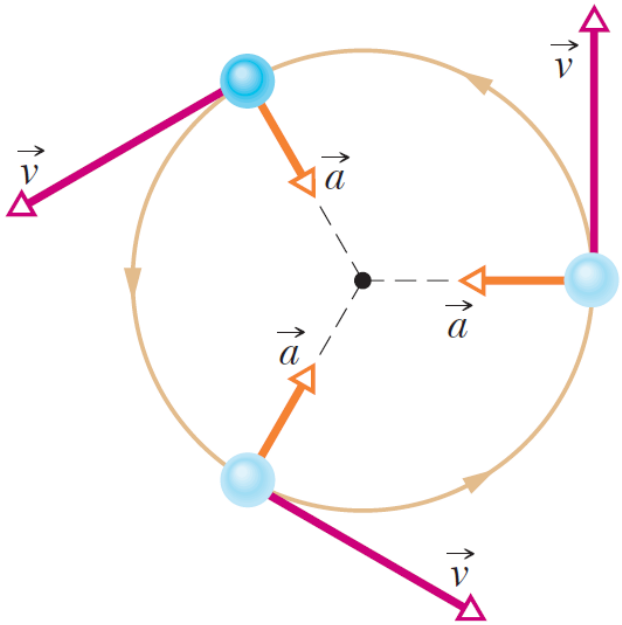
$$R = \frac{v_0^2}{g} \sin 2\theta_0.$$

$$x - x_0 = (v_0 \cos \theta_0)t.$$

$$y - y_0 = v_{0y}t - \tfrac{1}{2}gt^2$$

$$= (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2,$$

$$y = (\tan \theta_0)x - \frac{gx^2}{2(v_0 \cos \theta_0)^2}$$



$$a = \frac{v^2}{r}$$

$$T = \frac{2\pi r}{v}$$